UNIVERSITÄT ZU LÜBECK
INSTITUTE FOR SOFTWARE ENGINEERING
AND PROGRAMMING LANGUAGES

isp

State-Based Routing for Computational Sustainability

Introduction

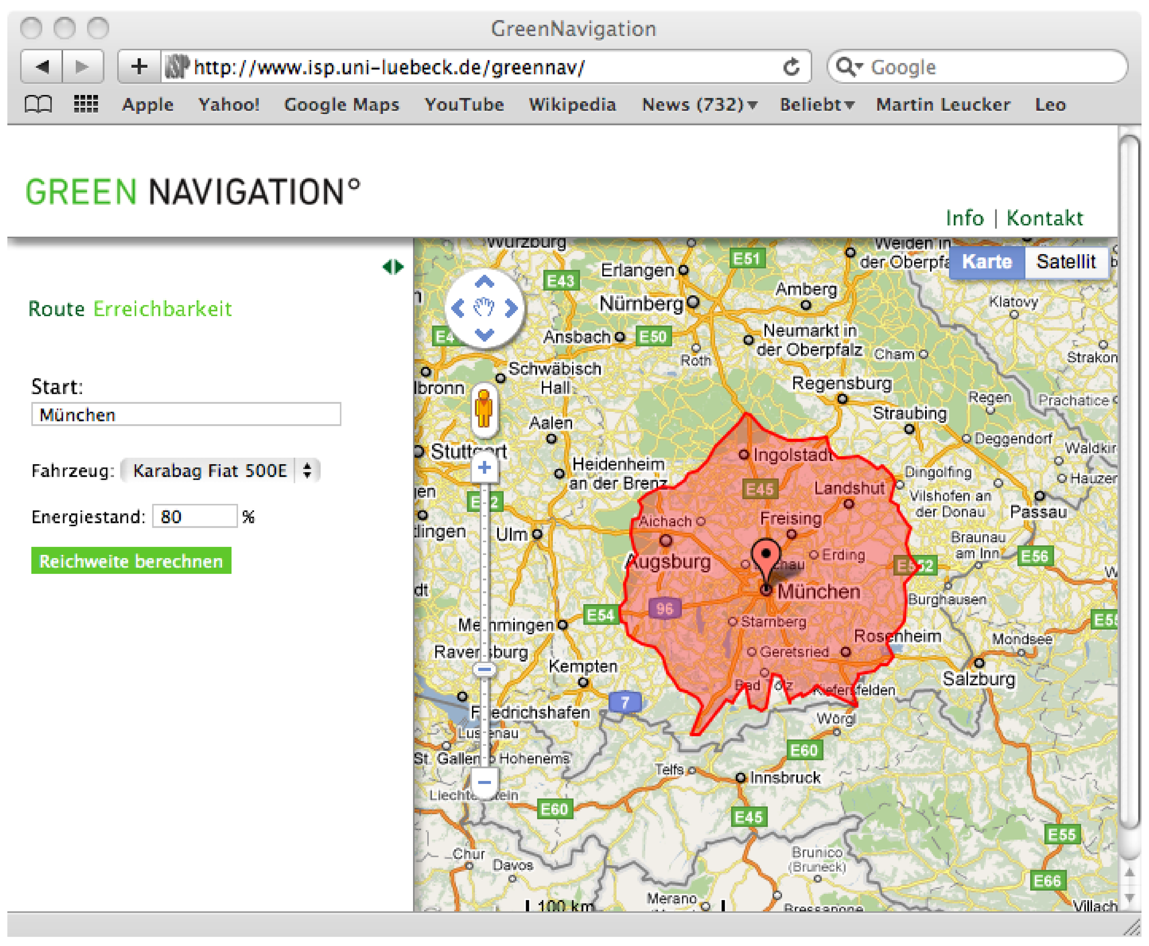
Various routing models each considering some aspects of finding "best" solutions:

- Shortest Path Problem: Finding a shortest path also yields an efficient path regarding energy consumption.
- Shortest Weight-Constrained Path Problem: Optimize more than one target function, e.g. time and energy-consumption.
- Time-Dependent and Multi-Modal Routing: Finding shortest paths depending on the time, caused for example by including public transportation.
- Energy-Optimal Routing: Considering energy constraints for electric vehicles.
- Stochastic Routing: Minimizing expected costs, maybe given certain conditional probabilities.
- Rerouting: After finding an efficient path and turning to a different direction (or gaining additional information), quickly find an alternate efficient route.

Each model comes with its own set of algorithms, our aim is to find a model unifying some aspects while still allowing for most algorithms known for the shortest path problem.

Prototype

The Technische Universität München (TUM) developed a prototypic routing service, which is further developed at the University of Lübeck, available at `www.isp.uni-luebeck.de/greennav`. It is used to evaluate different routing algorithms. You can see a range prediction of an electric vehicle.



Definition: State-Based Routing

- $G = (V, E)$ is a graph,
- S is a set of states preordered by $\leq_S$,
- $\mathcal{S} : V \to \mathcal{P}(S)$ describes possible states at each vertex,
- $\mathbb{W}$ is a set of *monotone* ($x_1 \leq_S x_2 \to f(x_1) \leq_S f(x_2)$) and *extensive* ($x \leq_S f(x)$) weights $S \rightsquigarrow S$,
- $\mathcal{W}' : E \to \mathbb{W}$ is a weighting,

such that

- $\mathcal{W}'(x, y)$ is a weight $\mathcal{S}(x) \to \mathcal{S}(y)$, and
- the *extension* of $\mathcal{W}'$ again is $\mathcal{W}$: walks $\to \mathbb{W}$ given by

$$\mathcal{W}(\gamma) = \mathcal{W}'(v_0, v_1) \circ \ldots \circ \mathcal{W}'(v_{k-1}, v_k)$$

for all walks $\gamma = (v_0, \ldots, v_k)$, $k \geq 0$ (identity for $k = 0$).

Objective:

- Given $x, y \in V$ and initial state $s \in \mathcal{S}(x)$, find at least one corresponding path for each minimal element in $\min(\mathcal{W}(\text{walks from } x \text{ to } y)(s))$ (except for equivalence), where

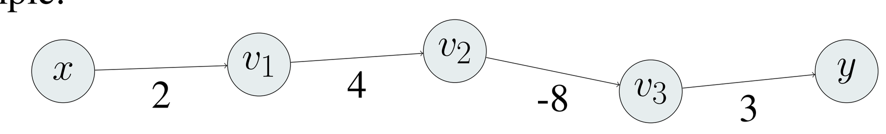
$$\min(S) := \{s \in S \mid \neg\exists s' \in S : s' < s\}.$$

Energy Constraints

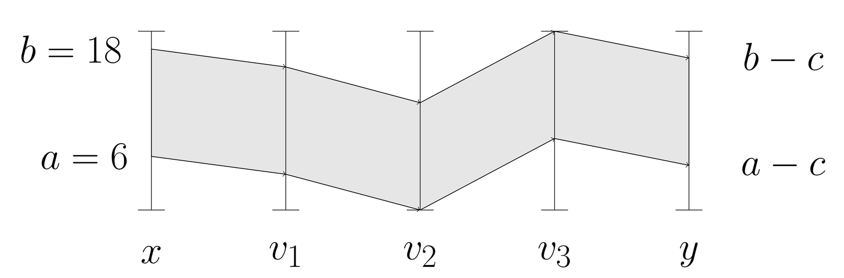
The set of states consists of battery charges in $B := [0, K] \cup \{-\infty\}$ (and altitudes):

- Maximum capacity $K > 0$
- Initial charge $J \in [0, K]$
- Recuperation (regaining energy)
- Can not drive with an empty battery

Example:



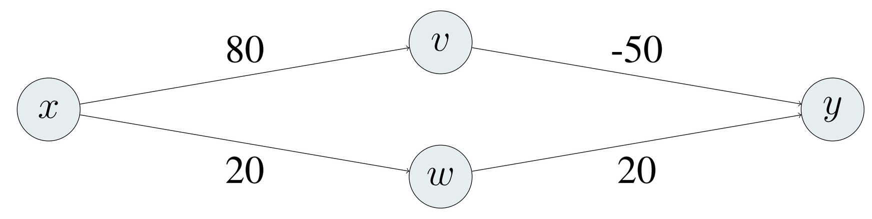
Battery transformation function of given path:



Battery charge more than $b = 18$ energy units are wasted. Having less than $a = 6$ energy units renders this path useless. The overall costs are $c = 2 + 4 - 8 + 3 = 1$.

State-Based Profile Search

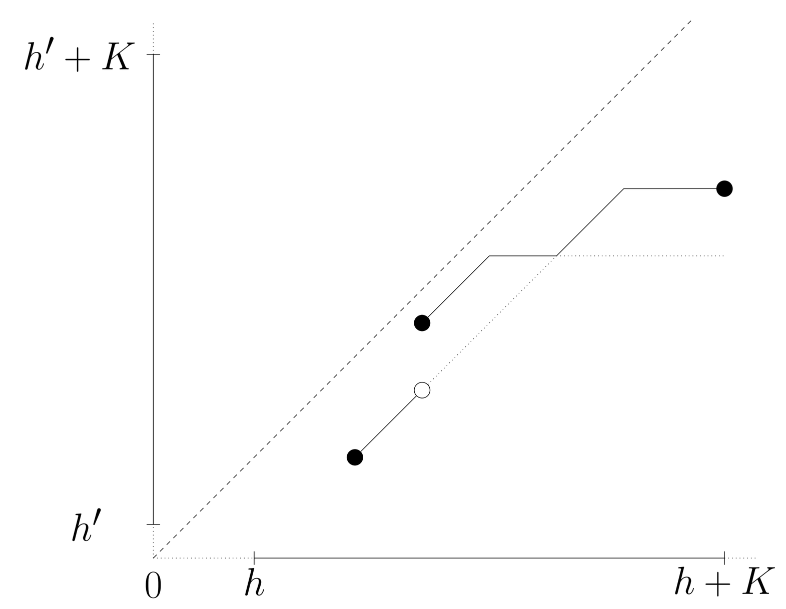
As it was done for the time-dependent routing problem, we may consider the problem of finding optimal solutions for each possible starting state. In terms of energy constraints, one path may be more efficient than another path, if able to invest a higher amount of battery charge.



- xvy less costs (30), but requires $J \geq 80$,
- xwy higher costs (40), but is possible with $J \geq 40$.

An optimal solution, called a *policy*, therefore maps battery charges $J \geq 80$ to xvy, all $40 \leq J < 80$ to xwy and all other J to no path.

Therefore, combining both functions by maximizing the energy value may yield a non-trivial cost function:



Relation to Time-Dependent Routing

Using a total order (or total preorder) the state-based routing problem is almost equivalent to time-dependent routing. Time-dependency is realized mainly by two changes to the common shortest path problem:

- Edge costs $f : \mathbb{R} \to \mathbb{R}_+$ are functions from departure times to time costs.
- FIFO property: $x_1 + f(x_1) \leq x_2 + f(x_2)$ for all $x_1 \leq x_2$.

Given an edge cost function f, we can define an appropriate g with $g(x) = x + f(x)$, i.e. considering the *arrival time*, such that

- g is extensive, because f is non-negative, and
- g is monotone due to the FIFO property.

Thus time-dependent routing is a special case of state-based routing.

Stochasticity

A direct generalization of energy-constraints using stochasticity is done by modeling the battery charge as a random variable

$$J : \Omega \to B \qquad (\text{with } B = [0, K] \cup \{-\infty\}).$$

The edge cost functions then are random variables itself $\Omega \to (B \to B)$, which can be formalized as a state transformation function as $(\Omega \to B) \to (\Omega \to B)$ using the same $\omega \in \Omega$.

The questionable decision is the choice of the partial preorder $\leq$. Two approaches are $J_1 \leq J_2$, if and only if

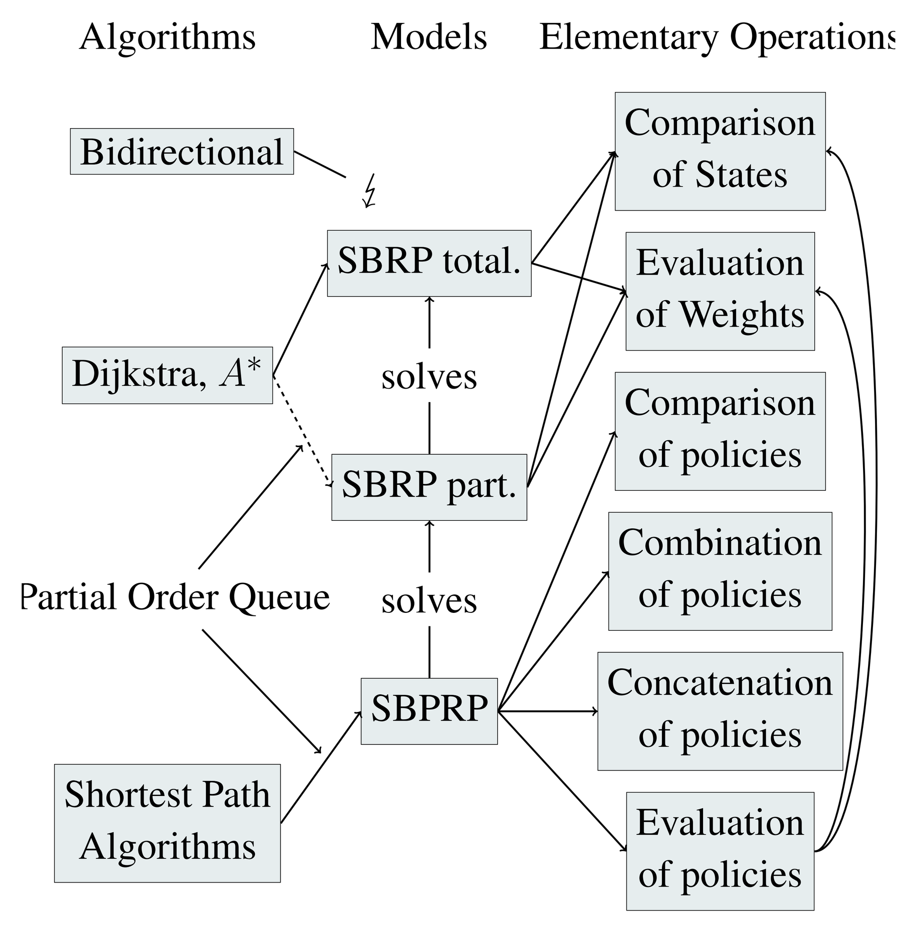
$$\mathbb{E}(J_1 \mid J_1 > -\infty) \geq \mathbb{E}(J_2 \mid J_2 > -\infty), \tag{1}$$
$$P(J_1 \geq c) \geq P(J_2 \geq c), \quad c \in [0, K]. \tag{2}$$

The former approach (1) yields non-monotone functions and thus does not fit our state-based approach. The latter (2) is an adaptation from Uludag et al. [14] and actually fits the state-based approach quite smoothly.

Algorithm Design

The computational problem here arises from the fact, that an optimal solution depends on the initial state. This renders a backward search impossible without doing a profile search. The same goes for almost all precomputation methods.

However, by considering the profile search, we may use existing algorithms with minor modifications as was shown for example by Eisner et al. [3]. The general concept for modifying existing algorithms is to use a partial order queue and to reconsider the stop condition.



In contrast to priority queues, partial order queues provide two essential functions:

- **min** - to find any minimal object, and
- **front** - to query the set of all minimal objects.

Conclusions and Future Work

First tests yielded reasonable results, but we just started to implement various shortest path algorithms for the state-based routing problem and its profile version. Hence runtime and space analyzations are near future goals. The model itself is promising, comprising time-dependent routing, battery constraints, stochasticity and multi-criteria routing, but the running time is an important question to answer soon.

In the long run we aim to extend GreenNav by more sophisticated algorithms and services. An egoistic routing algorithm is but the first step towards ecological sustainability. Multi-modality and stochasticity are two important points to consider.

Related Work

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[14] Suleyman Uludag, Ziyneti E. Uludag, Klara Nahrstedt, King-Shan Lui, and Fred Baker. A laplace transform-based method to stochastic path finding. In *Proceedings of the 2009 IEEE international conference on Communications*, ICC'09, pages 1319–1323, Piscataway, NJ, USA, 2009. IEEE Press.

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