State-Based Routing for Computational Sustainability

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1 Introduction

Green Routing is providing energy-efficient driving directions in order to promote ecological sustainability. It comprises various routing models, for example battery constraints [SLAH11], timedependency [DW09], stochasticity [UUN$^+$09], multi-modality [Paj09] and of course multi-criteria routing also known as the shortest weight-constrained path problem [Jok66]. This paper introduces state-based routing as an approach to unify some of these aspects, while still keeping most algorithms applicable.

2 Model

The concept of routing is to find an optimal path, where optimality is described by an ordering on real values describing for example the length of a path, the distance or the energy-consumption. The key insight is to generalize R to an arbitrary state space S and the order $\leq$ to an arbitrary preorder $\leq_S$ on S. The weights then become state transformation functions $S \to S$. In order to preserve some properties of the mentioned models, we need to restrict these transformation functions:

- The shortest path problem is defined for non-negative weights, in order to be able to apply Dijkstra's Algorithm [Dij59].
- In time-dependent routing, the presented functions need to fulfill the FIFO-property for the same reason [DW09].
- Routing with battery constraints is done by finding potentials in order to get non-negative energy-consumption, either by a Bellman-Ford algorithm or directly derived from altitude maps [SLAH11].

All in all, these properties are reflected by monotonicity ($a_1 \leq_S a2 \to f(a_1) \leq_S f(a_2)$) and extensivity ($a \leq_S f(a)$). Because by using a preorder $\leq_S$, a subset $A \subseteq S$ may have multiple minimal elements using he definition:

$$\min(A) := \{a \in A \mid \neg\exists b \in A : \ b <_S a\},$$

where $b <_S a$ is defined as $b \leq_S a \wedge \neg(a \leq_S b)$. Now, given vertices x, y and an initial state $s \in S$, the state-based routing problem is to find at least one corresponding path to each minimal resulting state except for equivalence. The state-based profile routing problem is to find an optimal profile, i.e. a mapping from initial states to optimal paths.

2.1 Example: Time-Dependency

In time-dependent routing the edge costs (time to traverse an edge) $c(t)$ are functions of the departure time t. The FIFO-property enforces $f(t_1)+t_1 \leq f(t_2)+t_2$ for $t_1 \leq t_2$. In a state-based perspective the departure time t is transformed to the arrival $g(t)$, where $g(t) := t + f(t)$. This way, we gain monotonicity from the FIFO-property and extensivity from having non-negative costs f.

Time-dependent routing is similar to state-based routing, it is a special case of our model. The crucial difference is, that we do not consider only time points but arbitrary states that are either partially or totally (pre-)ordered. That way, state-based routing comprises also other models, such as battery constraints.

2.2 Example: Battery Constraints

Routing with battery constraints was introduced by Sachenbacher et al. [SLAH11]. Instead of recursively defining energy path costs, we consider the battery transformation function of a single edge. Concatenation then is done by functional composition. The state space contains pairs of battery charges and potential energies $[0, K] \times R$ (where K is the battery capacity). An appropriate preorder is $(J_1, h_1) \leq (J_2, h_2)$ if and only if $J_1 + h_2 \geq J_2 + h_2$. Notice, that the preferred (minimal) states are those with a maximal energy level.

Eisner et al. [EFS11] have shown that Contraction Hierarchies (introduced by Geisberger [Gei08]) can be adapted to battery constraints by using multi-edge graphs. The profile search for state-based routing is generalizing this approach, such that multi-edge graphs are among the possible implementations of routing policies.

3 Conclusions

This paper introduced state-based routing, various routing models can be seen as special cases. In all its generality the described problem is NP-hard (by comprising the shortest-weight constrained path problem reducible to Partition [GJ90]), but depending on the state space, the ordering and the transformation functions the particular problems are as easy to solve as their original versions using the same algorithms.

Similarly to time-dependent routing, the edge cost functions are not fully specified. This is a top-down approach, such that algorithms solving the general problem of state-based routing can be specialized as soon as we gain additional information on the structure of the state space, the ordering and the transformation functions.

Our future work lies in implementing and evaluating the presented models using various algorithms and the combination of different models within the framework of state-based routing.

References

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