# Sensitivity Analysis of a Large Scale Integrated Land Use and Transportation System

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**Abstract.** In this work we perform sensitivity analysis of an integrated land use and transportation model. The modeling system used is TRANUS and the region modeled is the Grenoble urban agglomeration. The total effect of the input land use parameters on the *QoI* is investigated, which is the mean squared error in land use assignments. It is observed that 3 amongst 100 uncertain input parameters have maximum contribution towards *QoI* uncertainty.

## 1 Introduction

In a context of climate change and dwindling fossil resources, urban sustainability has become a key policy issue. Given the complexity of modern urban areas, designing sustainable policies calls for more than sheer expert knowledge. This is especially true for transport or land use policies, because of the strong interplay between the land use and the transportation systems. In such a context, land use and transport integrated modeling offers invaluable analysis tools for planners working on transportation and urban projects. Unfortunately the lack of confidence in their results, is somewhat nowadays an obstacle to a large diffusion to these tools. In particular, it is well known that uncertainty is pervasive in land use and transportation models. Uncertainty can be of epistemic kind, arising from mis-measurement, data mis-specification, and poor sampling. Also it can be of the aleatoric kind, which exhibits itself during forward propagation of the model. Quantification of these uncertainties is important for accurate forecasting of land use and transportation demands. It is a difficult problem to quantify uncertainty because of the large number of variables involved, and uncertainty within them is poorly characterized. Hence, it is important to identify major sources of uncertainty and their contribution towards output space variability. These reasons assert the need of performing sensitivity analysis while modeling a large scale land use and transportation system.

Uncertainty evolution has been studied by researchers for various ILUTMs, applied to model different regions [1,2]. The role of uncertainty in decision making and use parameter estimation in this case has also been investigated, by the land use community [3]. As far as sensitivity analysis is concerned, scientists have studied models involving few parameters in a small scale [4]. But an intensive analysis, focusing on sensitivity analysis of large scale ILUTM, with several parameters as uncertain, has largely been ignored.

In this work we have performed sensitivity analysis during calibration of the land use part of TRANUS [5], which was used to model the city of Grenoble, France. TRANUS land use algorithm consists in the resolution of a system of around 20 nonlinear equations and inequalities. The solution of such a system represents an economic equilibrium between supply and demand. The Grenoble land use model consists of 22 sectors which are spread amongst 225 zones. The total number of uncertain parameters for the land use model is 100. Each uncertain parameter has been assessed upon their *total effect* [6] on the *quantity of interest* or *QoI* , which is the $\mathcal{L}_2$ norm of the difference in calculated ($X$) and observed production ($X^{obs}$).

$$Q = \left|\left|X^{obs} - X\right|\right|_2 \tag{1}$$

## 2 Methodology

The 22 economic sectors in the Grenoble model consist of population classified into 7 categories, employment classified into 8 categories, and real estate classified into 7 categories which interact with each other; see Table 1 or [7]. The model describes all the interactions via a set of parameters which are then increased by the number of interactions. In the current work, the parameters assumed to be uncertain are the maximum and minimum consumption amounts, as well as the elasticity parameters in the induced demand function; the penalization parameters and substitution elasticities in the substitute demand equation, and the utility level in the locational utility function. Referring to TRANUS mathematical description [8], the uncertain parameter vector is given by

$$\Psi = [\min^{mn}, \max^{mn}, \delta^{mn}, \bar{\omega}^{mn}, \delta, \theta^{n}]^{\top},$$

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where, $\Psi \in \mathbb{R}^{100}$; the index $m$ and $n$ covering all economics sectors. Due to interactions amongst themselves, these parameters are constrained. Sampling such a space, is difficult, especially when the dimensionality is high [9]. Hence the sensitivity analysis problem was computationally hard.

We performed a complexity analysis of the algorithm for land use part of TRANUS. Given $N$ zones and $M$ sectors, the total production has an order of complexity of $\mathcal{O}(N^4M^2)$. If the convergence is achieved after $\mathcal{Z}$ iterations, then the overall complexity of the total production computation is $\mathcal{O}\left(\mathcal{Z} \times N^4M^2\right)$. For a sensitivity analysis using $P_s$ samples for each parameter, each having sample size $R_s$, the overall complexity for production is $\mathcal{O}\left(P_sR_s \times \mathcal{Z} \times N^4M^2\right)$ for each parameter. The order then increases polynomially as the number of zones and sectors increase.

In this work, we have employed high performance computing, and use of parallel machines to perform sensitivity analysis. We have used symmetric multiprocessing [10] to distribute TRANUS implementation in $P_sR_s$ parallel machines. However, the scalability of the land use algorithm itself was not investigated in the current work. In future, our plan is to further parallelize the execution of TRANUS land use model.

**Table 1.** Description of the sectors.

| Sector number | Description | Sector Type |
|---|---|---|
| 1 | Industrial employment | Employment |
| 2 | Public employment | Employment |
| 3 | Offices, research& development employment | Employment |
| 4 | Retail employment | Employment |
| 5 | School employment | Employment |
| 6 | Household income (rich) | Income |
| 7 | Household income (above average) | Income |
| 8 | Household income (below average) | Income |
| 9 | Household income (poor) | Income |
| 10 | Students | Income |
| 11 | Individual housing | Real estate |
| 12 | Apartment Housing | Real estate |
| 13 | Land for economic activities | Real estate |
| 14 | Land for shops in urban area | Real estate |
| 15 | Social housing | Real estate |
| 16 | University | Employment |
| 17 | Retail employment (less frequent) | Employment |
| 18 | Supermarket employment | Employment |
| 19 | Retired income (rich) | Income |
| 20 | Retired income (poor) | Income |
| 21 | Other commercial ventures | Employment |
| 22 | Other commercial land use | Real estate |

## 3 Results

To carry out the sensitivity analysis, Monte Carlo method is used [11]. As little or no prior information about the input parameters are available, the joint density is assumed to be the product of independent Gaussian marginal densities. The Monte Carlo samples for sensitivity analysis are obtained using rejection sampling on the joint density [12], taking into account the constraints amongst parameters.

At first, total effect of the input parameters on $QoI$ in eqn. (1) is studied. It is observed that, the minimum amount of *land used for economic activities* required by *industry* ($\min^{1-13}$) has 81% total effect on the $QoI$, followed by elasticity parameter of *below average income households* due to cost of *individual housing* $\delta^{8-11}$ at 14%. It is described by a pie chart in fig. 1. Rest of the parameters have negligible or lesser effect on $QoI$.

We observe similar behaviors as previously observed, if we consider the price and the production convergence indicators (see [8]) as $QoI$s instead of the $\mathcal{L}_2$ production error (1). For both cases only one parameter, i.e., $\delta^{8-11}$, is responsible for majority of the unexplained output variation, see fig. 2(a) and fig. 2(b).

An analysis, of the total effect when only one kind of parameter is uncertain; is also performed. It is observed that if only elasticity parameters or minimum or maximum consumption amounts are varied, $\delta^{8-11}$, $\min^{1-13}$ and $\max^{1-13}$ have the maximum total effect on the $QoI$ , respectively. On the contrary, if the parameters of the demand substitution function, i.e. substitution elasticity ($\delta$) or penalty ($\bar{\omega}^{mn}$) parameters and utility level parameter ($\theta^n$), are assumed to be uncertain, then more than one parameter contribute significantly, towards $QoI$ variance. Figure 3 show the plots.

As far as the computational time is concerned, one iteration of TRANUS calibration step required approximately 530 seconds. Considering 100 samples of sample size 100 each the total sensitivity analysis time, if performed serially would take 1472.22 hours for a single parameter. We used 100 parallel machines which decreased the processing time 100 times.

## 4 Conclusion

In this work, we have performed sensitivity analysis during the calibration of the TRANUS model applied to the city of Grenoble in France (only on the land use part). The uncertain parameters taken were parameters from demand and substitution functions. It was observed that the unexplained variance in the quantity of interest, mainly due to parameters $\min^{1-13}$, $\delta^{8-11}$ and $\max^{1-13}$, in that order. In future, we plan to use this information to solve a inverse problem and perform parameter estimation during calibration of TRANUS model of Grenoble.

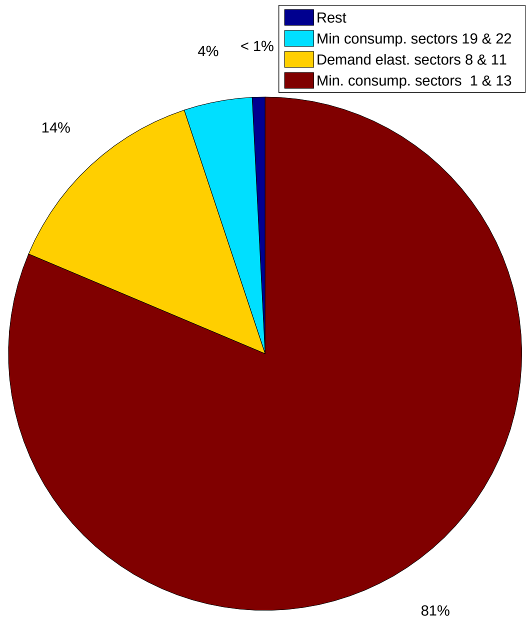
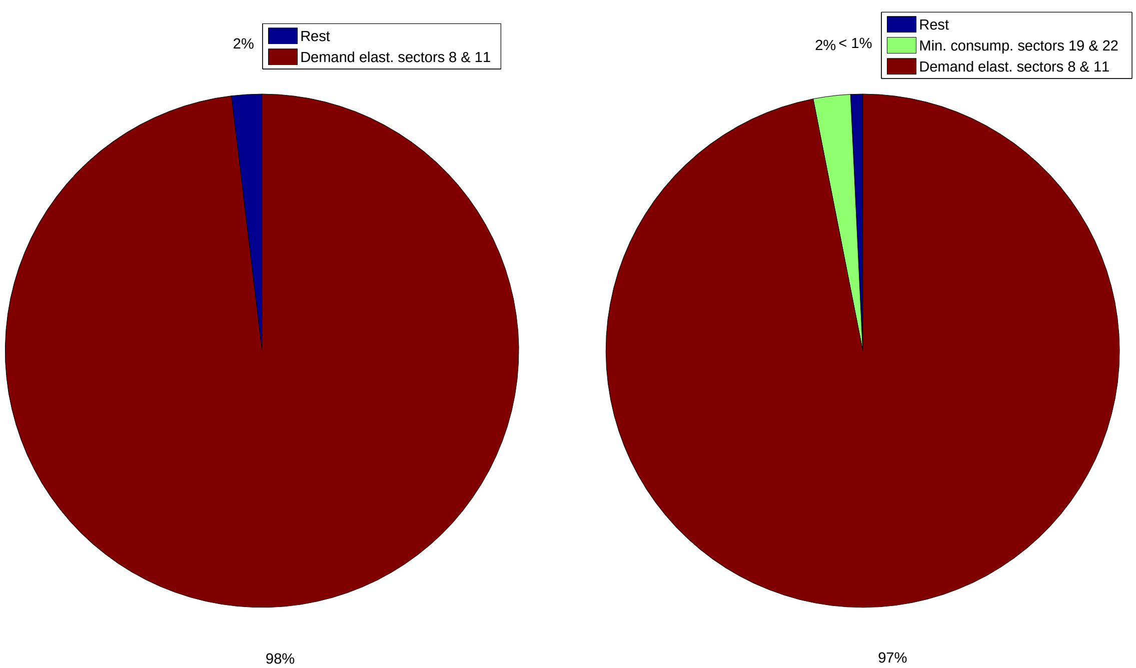


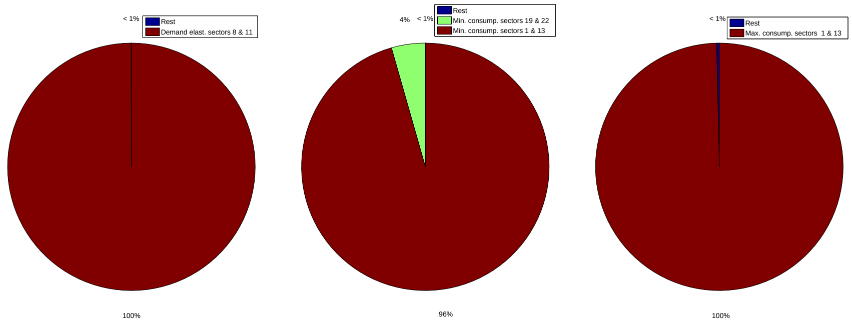

**Fig. 1.** Total effect of parameters on the $QoI$ . Sector 8 and 11 refers to *'below average income households'* and *'individual housing'* and 1 and 13 refers to *'industrial employment'* and *'land for economic activities'*



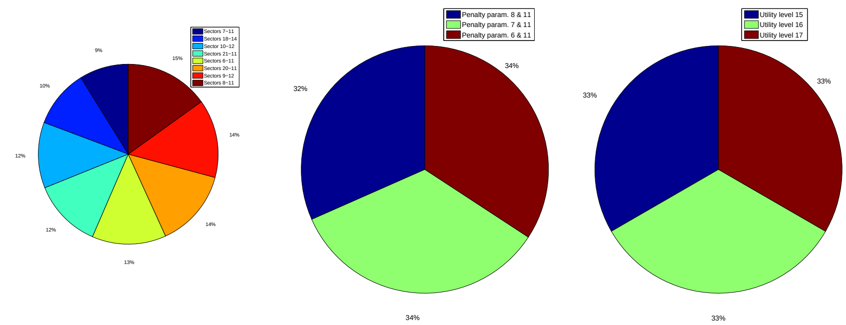

(a) The production convergence indicator is the *quantity of interest*

(b) The price convergence indicator is the *quantity of interest*

**Fig. 2.** Description of total effect when a) production and b) price convergence indicator are the $QoI$s.




(a) Demand elasticity $\delta^{mn}$. (b) Minimum consumption $\min^{mn}$. (c) Maximum Consumption $\max^{mn}$.




(d) Demand substitution elasticity $\bar{\omega}^{mn}$. (e) Penalty parameter $\delta$. (f) Utility level $\theta^n$

**Fig. 3.** Total effect when only certain type of variable is uncertain. Top row represents parameters of the elasticity function. Notice the dominance of few parameters towards $QoI$ uncertainty. Bottom row represents parameters of the substitution function and utility level. The effects are distributed evenly amongst parameters.

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