Unité BIA Toulouse
Biométrie et Intelligence Artificielle

# Model-based adaptive spatial sampling for occurrence map construction

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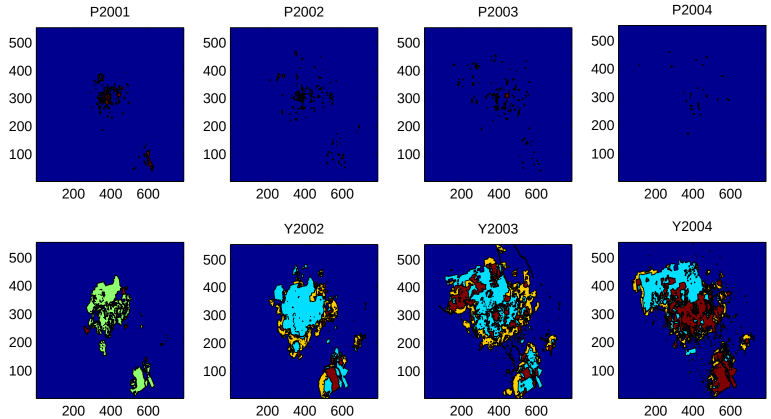
# Mapping spatial processes in environmental management

## Mapping pest occurrence

- Building pest occurrence map in order to eradicate
- Observations costly
- Errors in mapping also costly



# Mapping spatial processes in environmental management

**Different problems depending on observations nature**

- Data visualization
  - Complete observations (everywhere)
  - Perfect observations (No errors/missing data)

  $\Rightarrow$ How to visualize data?
- Map reconstruction
  - Complete observations
  - Noisy observations

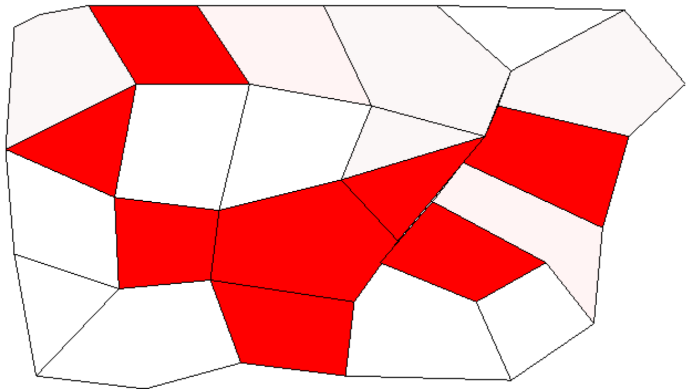
  $\Rightarrow$ How to reconstruct the “true” map?
- **Sampling and map construction**
  - Incomplete observations (not everywhere)
  - Noisy observations

  $\Rightarrow$ Where to observe? / How to reconstruct?

# Mapping spatial processes in environmental management

**How to design an efficient spatial sampling method to estimate an occurrence (0/1) map when**

- ✓ process to map has spatial structure
- ✓ observations are imperfect/incomplete
- ✓ sampling is costly
- ✓ process does not evolve during the sampling period



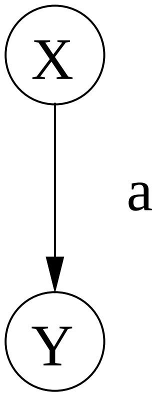
# Overview of the proposed approach

**Optimization approach for designing spatial sampling policies**

The Hidden Markov Random Field model is used for:

- Representing current uncertain knowledge about map to reconstruct
- Updating knowledge after observations
- Defining a unique criterion for
  - map reconstruction from observed data
  - spatial sampling actions selection

# Optimal sampling problem




Hidden variable $X$

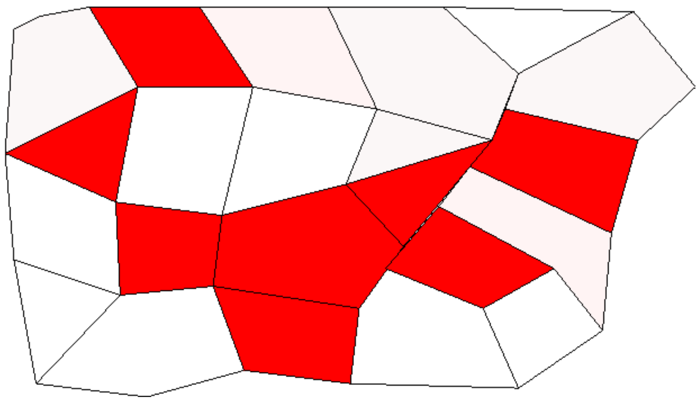
Sampling action $a$

Observation model
$p(Y = o|x, a)$

**Question**: How to reconstruct hidden variable $X$ using sampling actions?

1. Hidden variable model
2. Updated model after sampling result
3. Hidden variable reconstruction
4. Sampling action optimization

# Spatial sampling optimization



The hidden variable $x$ is a map

$\Rightarrow$ The sampling optimization problem has to be revisited

**Question**: How to reconstruct hidden map $x$ using sampling actions?

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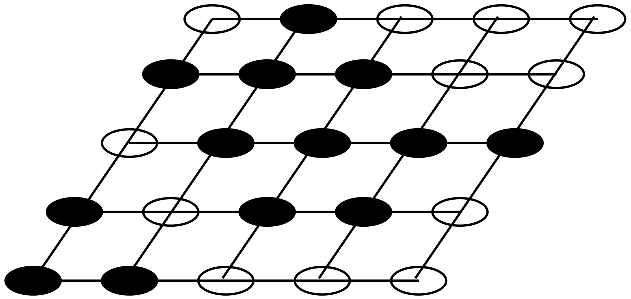
# Pairwise Markov random field (1)



- Multiple interacting variables
- Independence given neighborhood

$\Rightarrow$ Pairwise Markov random field

**Question**: How to reconstruct hidden map $x$ using sampling actions?

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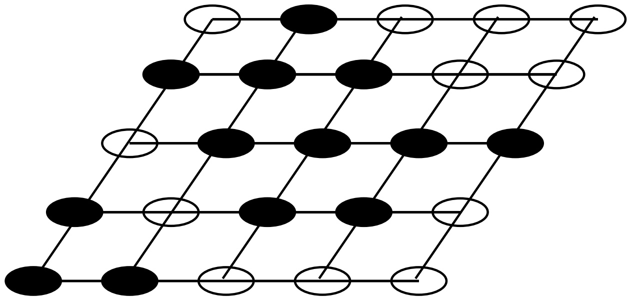
# Pairwise Markov random field (2)



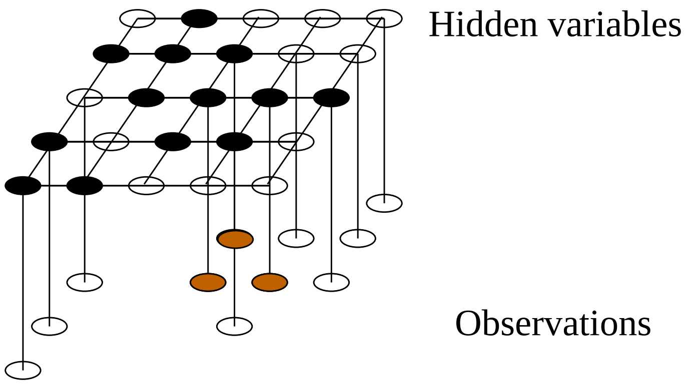
- Multiple interacting variables
- Independence given neighborhood

$\Rightarrow$ Pairwise Markov random field

- Interaction graph $G = (V, E)$
- $\psi_i$: “weights” on states of vertex $i$
- $\psi_{ij}$: correlations “strength” between neighbor vertices
- $Z$: normalizing constant / partition function

$$P(x) = \frac{1}{Z}\Big(\prod_{i \in V} \psi_i(x_i)\Big)\Big(\prod_{(i,j)\in E} \psi_{ij}(x_i, x_j)\Big)$$

# Hidden Markov random field (1)




- $a \in \{0,1\}^{|V|}$: subset of $V$ selected for sampling
- Independent observations:

$$P(o|x,a) = \prod_{i \in V} P_i(o_i|x_i,a_i)$$

**Question**: How to reconstruct hidden map $x$ using sampling actions?

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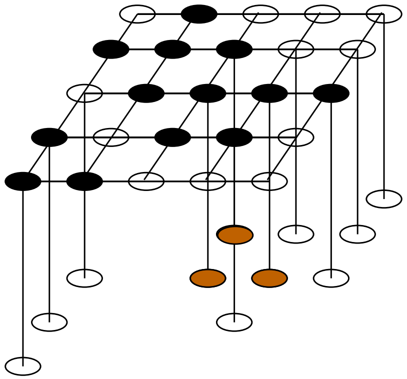
# Hidden Markov random field (2)




- $a \in \{0,1\}^{|V|}$: subset of $V$ selected for sampling
- Independent observations:

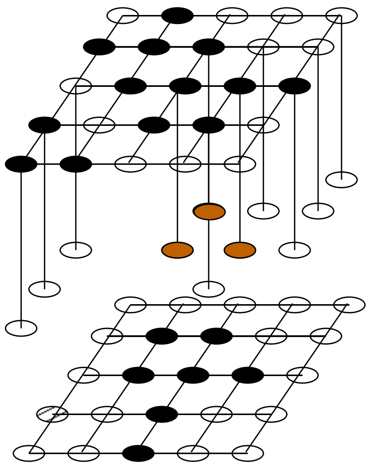
$$P(o|x,a) = \prod_{i \in V} P_i(o_i|x_i,a_i)$$

## Updated Markov random field (Bayes' theorem)

$$P(x|o,a) = \frac{1}{Z}\Big(\prod_{i \in V} \psi'_i(x_i,o_i,a_i)\Big)\Big(\prod_{(i,j)\in E} \psi_{ij}(x_i,x_j)\Big) \text{ where}$$

$$\psi'_i(x_i,o_i,a_i) = \psi_i(x_i)P_i(o_i|x_i,a_i)$$

# Hidden map reconstruction (1)




Local (MPM):

$$x_i^* = \arg\max_{x_i} P_i(x_i|o,a), \forall i \in V$$

**Question**: How to reconstruct hidden map $x$ using sampling actions?

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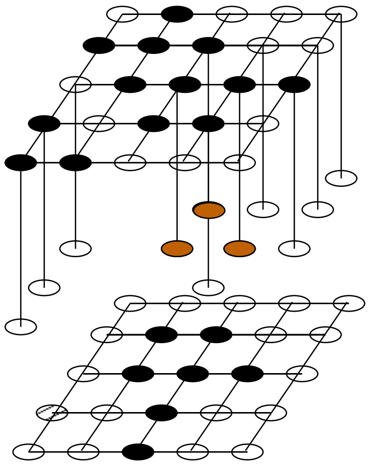
# Hidden map reconstruction (2)




Local (MPM):

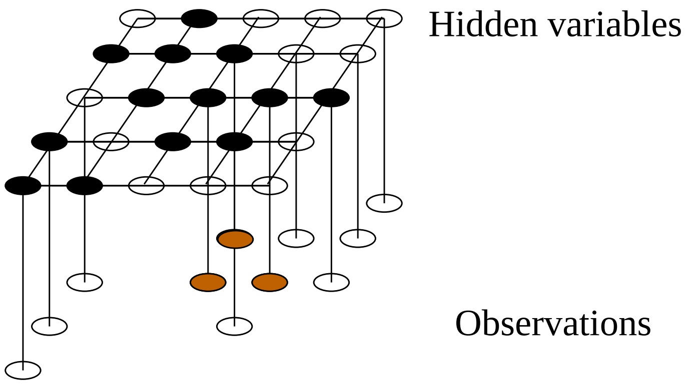
$x_i^* = \arg\max_{x_i} P_i(x_i|o,a)$

**Value of reconstructed map**

Expected number of well classified sites in $x^*$

$$V^{MPM}(o,a) \quad = \quad f\Big(\sum_{i\in V} \max_{x_i} P_i(x_i|o,a)\Big)$$

# Sampling action optimization (1)




- $a \in \{0,1\}^{|V|}$ selected for sampling
- Independent observations $o \in \{0,1\}^{|V|}$

$\Rightarrow$ How to optimize the choice of $a$?

**Question**: How to reconstruct hidden map $x$ using sampling actions?

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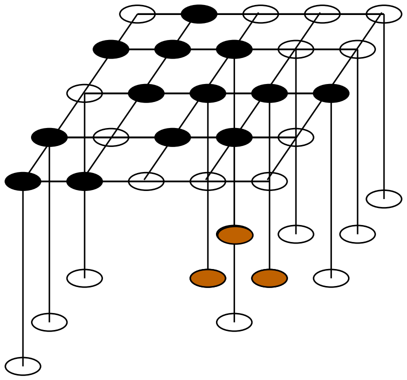
# Sampling action optimization (2)




- $a \subseteq V$ selected for sampling
- Independent observations $o$ result

$\Rightarrow$ How to optimize the choice of $a$?

$$U(a) = -c(a) + \sum_o P(o|a)V(o,a)$$
$$a^* = \arg\max_a U(a)$$

- The computation of $a^*$ is hard! (NP-hard)
- Only feasible for small problems or needs approximation!

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# Approximate spatial sampling (1)

Approximate the computation of

$$a^* = \arg\max_a -c(a) + \sum_o P(o|a) V^{MPM}(o,a)$$

- Explore cells where initial knowledge is the most uncertain: marginal $P_i(x_i|o,a)$ closest to $\frac{1}{2}$

$$\tilde{a} = \arg\max_a -c(a) + f\left(\sum_{i,a_i=1} \min\left\{P_i(X_i=1), P_i(X_i=0)\right\}\right)$$

- Marginals computation is itself *NP-hard*
$\Rightarrow$ approximation using belief propagation (sum prod) algorithm

# Approximate spatial sampling (2)

The approximation results from simplifying assumptions:

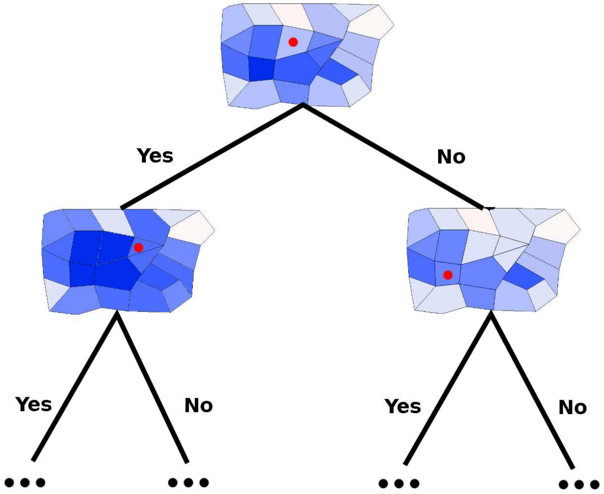
- Sampling actions are reliable
- No passive observations
- Joint probability approximated by one with idependent factors

# Adaptive spatial sampling (1)

- Idea:
  - Sampling locations not chosen once for all before the sampling campaign
  - Intermediate observations are taken into account to design next sampling step
  - Possibility to visit a cell more than once

# Adaptive spatial sampling (2)

- a sampling strategy $\delta$ is a tree
- a trajectory in $\delta$: $\tau = (a^1, o^1, \dots, a^K, o^K)$




**Value of a leaf**

$$U(\tau) = -\sum_{k=1}^{K} c(a^k) + V^{MPM}(o^0, o^1, \dots, o^K, a^0, a^1, \dots, a^K)$$

**Value of a strategy** $V(\delta) = \sum_\tau U(\tau) P(\tau \mid \delta)$

# Heuristic adaptive spatial sampling

- Exact computation is *PSPACE-hard* !

⇒ Heuristic algorithm

- on line computation
- approximate method for static sampling at each step

# Concluding remarks

- A framework for spatial sampling optimization:
  - based on Hidden Markov random fields
  - different map quality criteria
  - extended to "adaptive" sampling
- Problems too complex for exact resolution

  $\Rightarrow$ Heuristic solution based on approximate marginals computation
- Empirical validation on simulated problems:
  - Comparison of SSS, ASS and classical sampling methods (random sampling, ACS)
  - Markov random fields parameters learned from real data
  - ASS $>$ SSS $>$ classical methods

# Ongoing work

- Exact algorithms for small problems (Usman Farrokh): combining variable elimination and tree search
- “Random sets + kriging” approach (Mathieu Bonneau): development of a dedicated approximate method and comparison to the HMRF approach
- PhD thesis on *adaptive spatial sampling for weeds mapping at the scale of an agricultural area* (Sabrina Gaba, INRA-Dijon).
- Future?

$\Rightarrow$ Spatial partially observed Markov decision processes

## Questions?

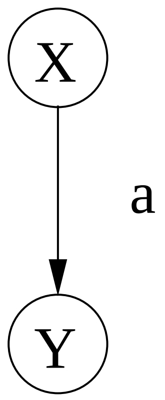
# Thanks for listening

# Contents

1- Optimal sampling of a hidden random variable
2- Defining optimal spatial sampling problems
3- Approximate computation of an optimal strategy
4- Evaluation of proposed method on simulated data

# Optimal sampling problem

## Hidden variable model




Prior model $P(x)$

**Question**: How to reconstruct hidden variable $X$ using sampling actions?

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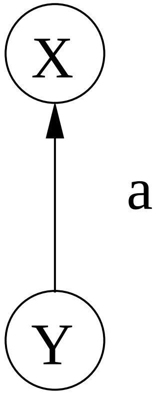
# Optimal sampling problem

## Updated model




Posterior: $P(x|o,a) = \dfrac{P(o|x,a)P(x)}{P(o|a)}$

**Question**: How to reconstruct hidden variable $X$ using sampling actions?

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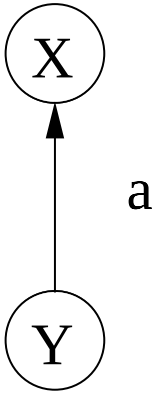
# Optimal sampling problem

## Hidden variable reconstruction




$$x^*(o,a) = \arg\max_x P(x|o,a)$$
$$V(o,a) = f(P(x^*|o,a))$$

**Question**: How to reconstruct hidden variable $X$ using sampling actions?

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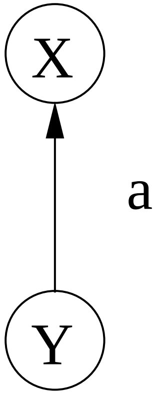
# Optimal sampling problem

## Hidden variable reconstruction



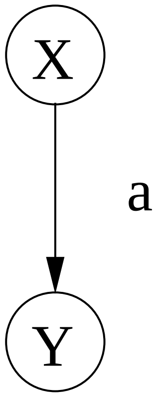

$$x^*(o,a) = \arg\max_x P(x|o,a)$$
$$V(o,a) = f(P(x^*|o,a))$$

**Question**: How to reconstruct hidden variable $X$ using sampling actions?

- $x^*(o,a)$ is the best reconstruction given sampling result $(o,a)$
- $V(o,a)$ is the value of reconstructed variable after sampling result $(o,a)$

# Optimal sampling problem

## Sampling action optimization




$$U(a) = -c(a) + \sum_o P(o|a)V(o,a)$$

$$a^* = \arg\max_a U(a)$$

**Question**: How to reconstruct hidden variable $X$ using sampling actions?

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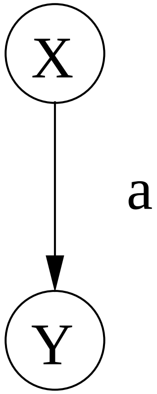
# Optimal sampling problem

## Sampling action optimization




$$U(a) = -c(a) + \sum_o P(o|a)V(o,a)$$
$$a^* = \arg\max_a U(a)$$

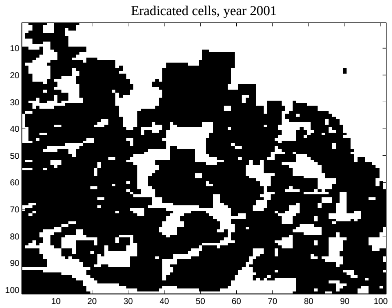
**Question**: How to reconstruct hidden variable $X$ using sampling actions?
The value of an action is a tradeoff between

- The cost $c(a)$ of the action and
- The expected quality of the reconstructed variable (over all possible sample results)

# Contents

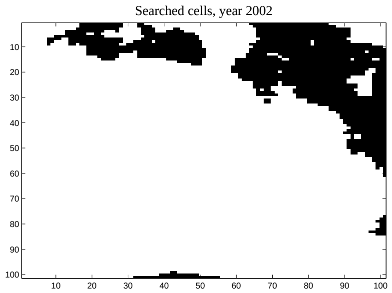
# Contents

# HMRF model for fire ants problem (1)



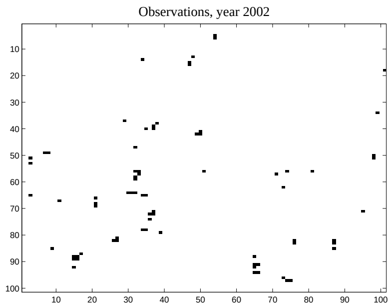

Eradication

$(e)$




Search actions

$(a)$




Observations

$(o)$

- eradication (at previous year): $e_i \in \{0, 1\},\ i = 1, \dots n$
- search actions: passive search or active search, $a_i \in \{0, 1\},\ i = 1, \dots n$
- observations: no nest detected / at least one nest detected, $o_i \in \{0, 1\},\ i = 1, \dots n$

# HMRF model for fire ants problem (2)

- Distribution on maps = Potts model

$$P_e(x \mid \alpha, \beta) = \frac{1}{Z} \exp\Big( \sum_{i \in V} \alpha_{e_i} \operatorname{eq}(x_i, 1) + \beta \sum_{(i,j) \in E} \operatorname{eq}(x_i, x_j) \Big)$$

- Distribution of observation given map, $P_{a_i}(o_i \mid x_i, \theta)$

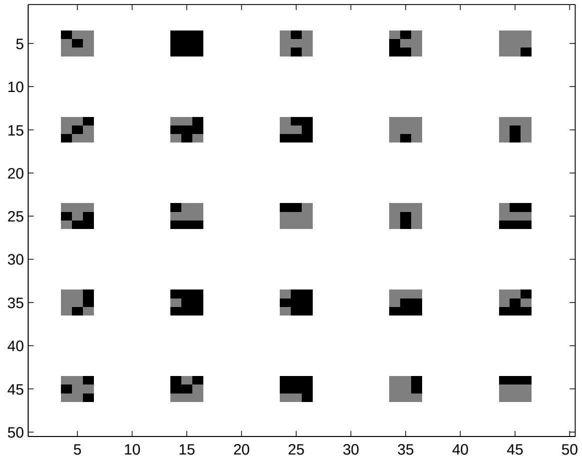
| $o_i \setminus x_i$ | 0 | 1 |
|---|---|---|
| 0 | 1 | $1 - \theta_{a_i}$ |
| 1 | 0 | $\theta_{a_i}$ |

with $\theta_0 < \theta_1$

# HMRF model for fire ants problem (3)

An initial arbitrary sampling $(a^0, o^0)$ is used for:

- Parameters estimation: $\lambda = (\alpha, \beta, \theta)$
  approximate version of EM for HMRF (Simul field EM)
  - identification problem between $\alpha$ and $\theta$
  - OK if $\theta$ known: use of expert values
- Marginals computation: $P_i(x_i | o_i^0, a_i^0)$



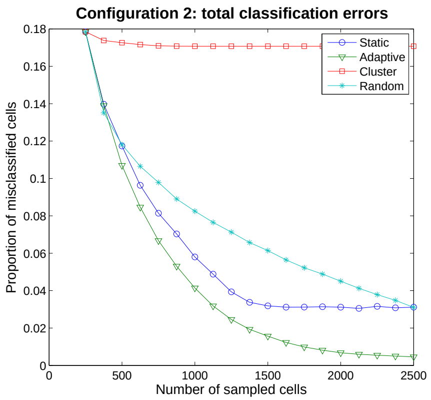
# Heuristic sampling methods evaluation (1)

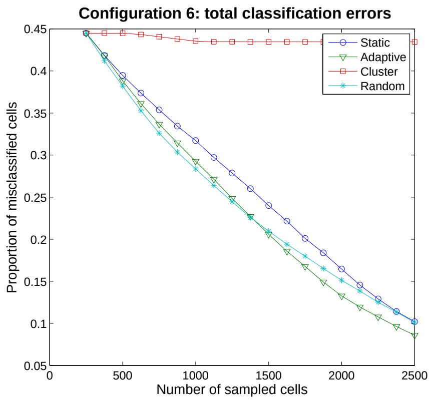
- Evaluation on simulated data
- Comparison of behavior of
  - random sampling (RS)
  - adaptive cluster sampling (ACS)
  - static heuristic sampling (SHS)
  - adaptive heuristic sampling (AHS)

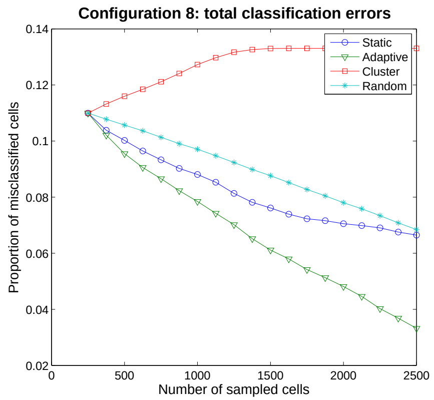
# Heuristic sampling methods evaluation (2)

- Procedure: repeat 10 times
  - simulate hidden map $x$ from $P(x \mid \alpha, \beta)$ ($50 \times 50$ cells)
  - apply regular sampling (about $10\%$ of area): $a^0$
  - simulate $o^0$ from $P_{a_i}(o_i \mid x_i, \theta)$ (regular sampling plus passive search)
  - estimate initial knowledge
  - apply RS, ACS, SHS, AHS, 10 times

# Rate of misclassified cells




$\alpha = (0, -2), \beta = 0.8 \quad \alpha = (0, 0), \beta = 0.5 \quad \alpha = (1 - 1), \beta = 0.4$

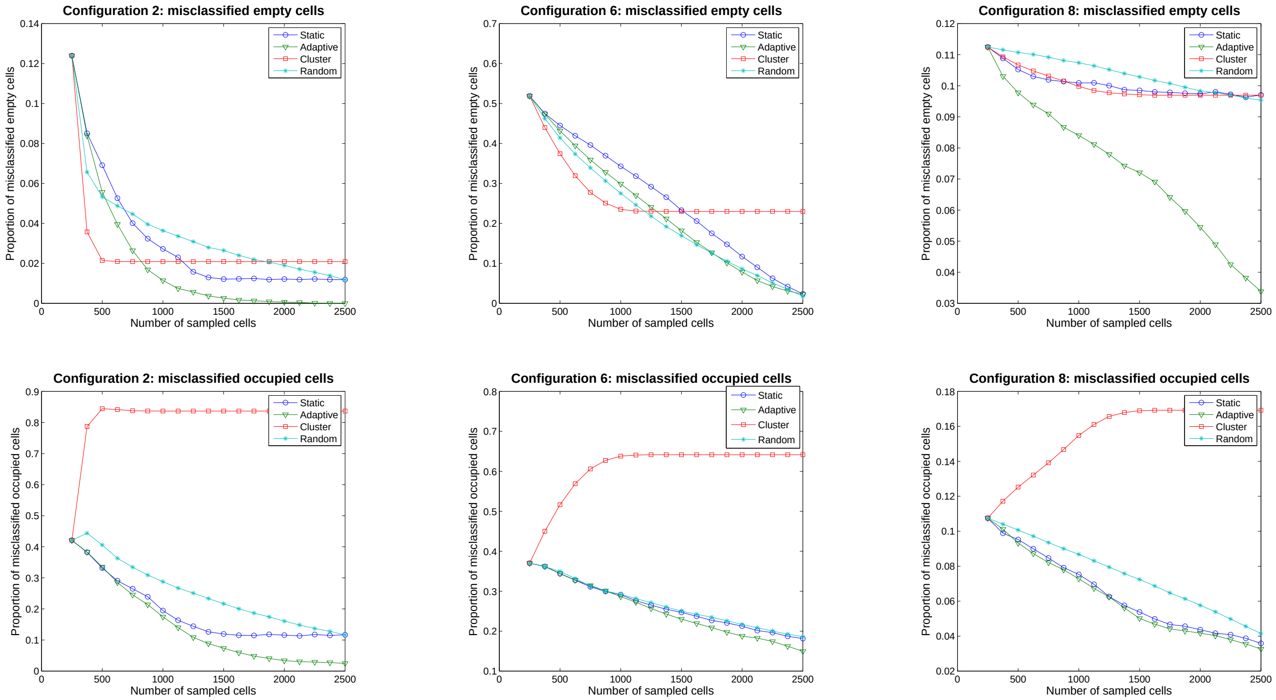
$$\theta = (0, 0.8)$$

legend: SHS AHA ACS RS

# Per color error rate

misclassified
empty cells

misclassified
occupied
cells




$$\alpha = (0, -2) \quad \alpha = (0, 0) \quad \alpha = (1 - 1)$$

$$\beta = 0.8 \quad \beta = 0.5 \quad \beta = 0.4$$

egend:

SHS AHA ACS

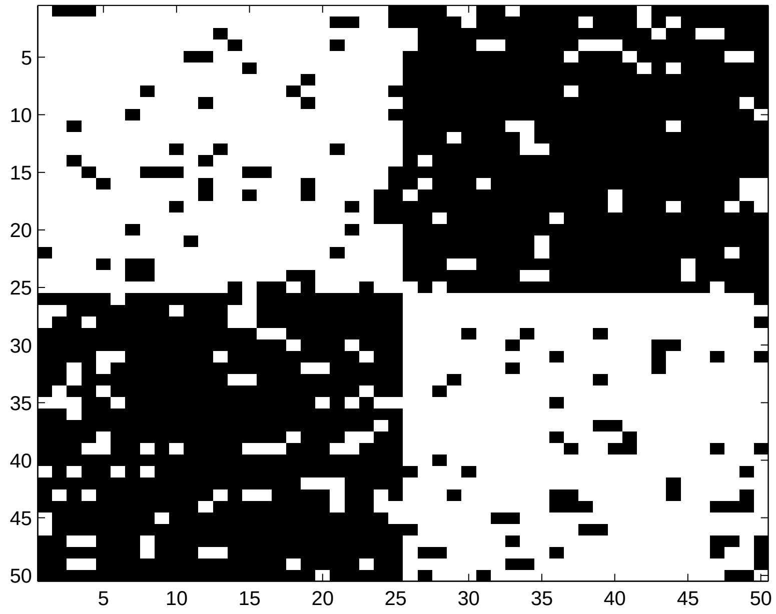
RS

# General behavior

- ACS is not adapted (as expected): poor results
- Adaptive HS $\geq$ Static HS $\geq$ Random S
- Discrepancy between Adaptive HS and Static HS increases with
  - sampling ressources
  - hidden map structure

# Where do we sample?

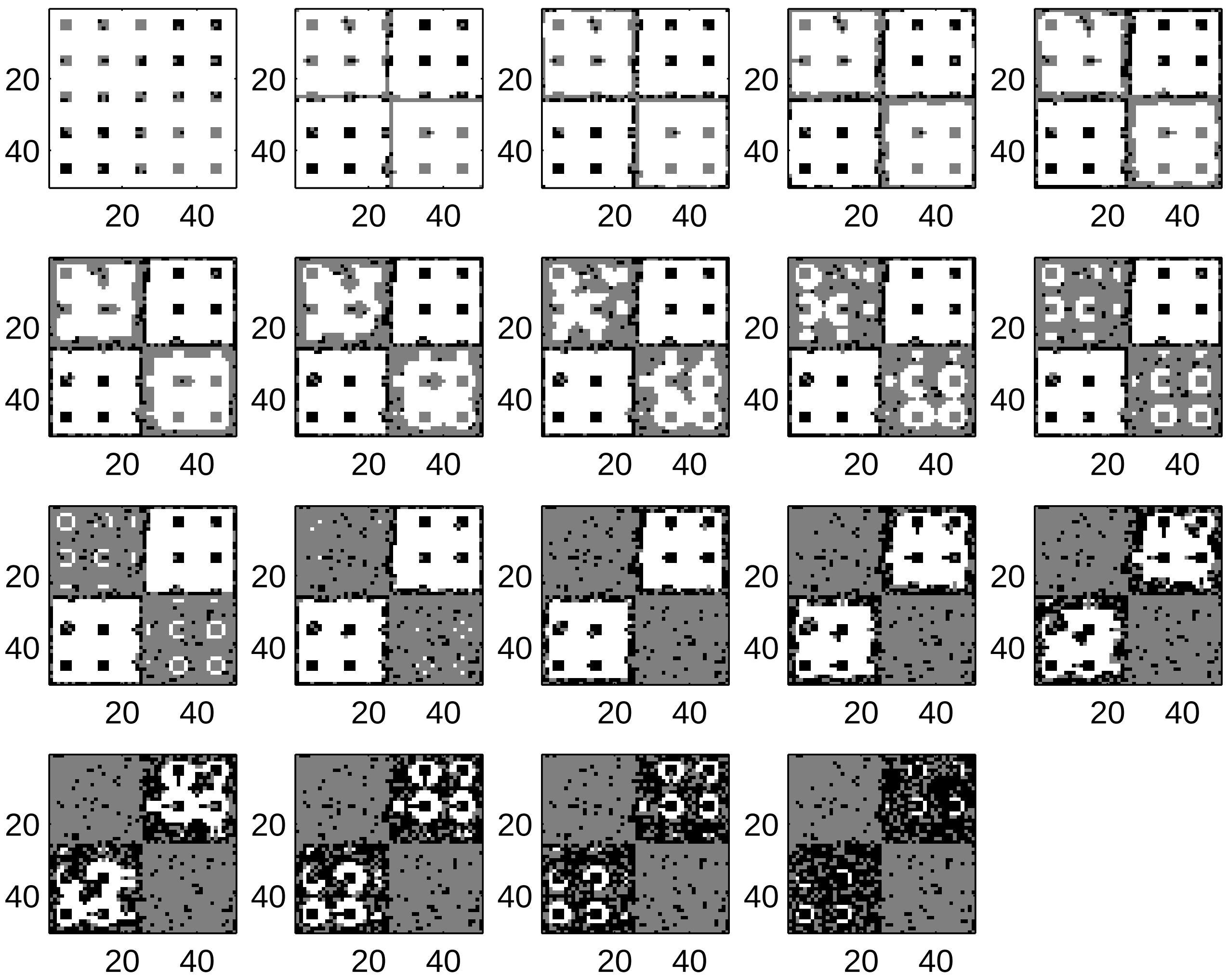
## Hidden map




$$\alpha = (1, -1), \beta = 0.4, \theta = (0, 0.8)$$

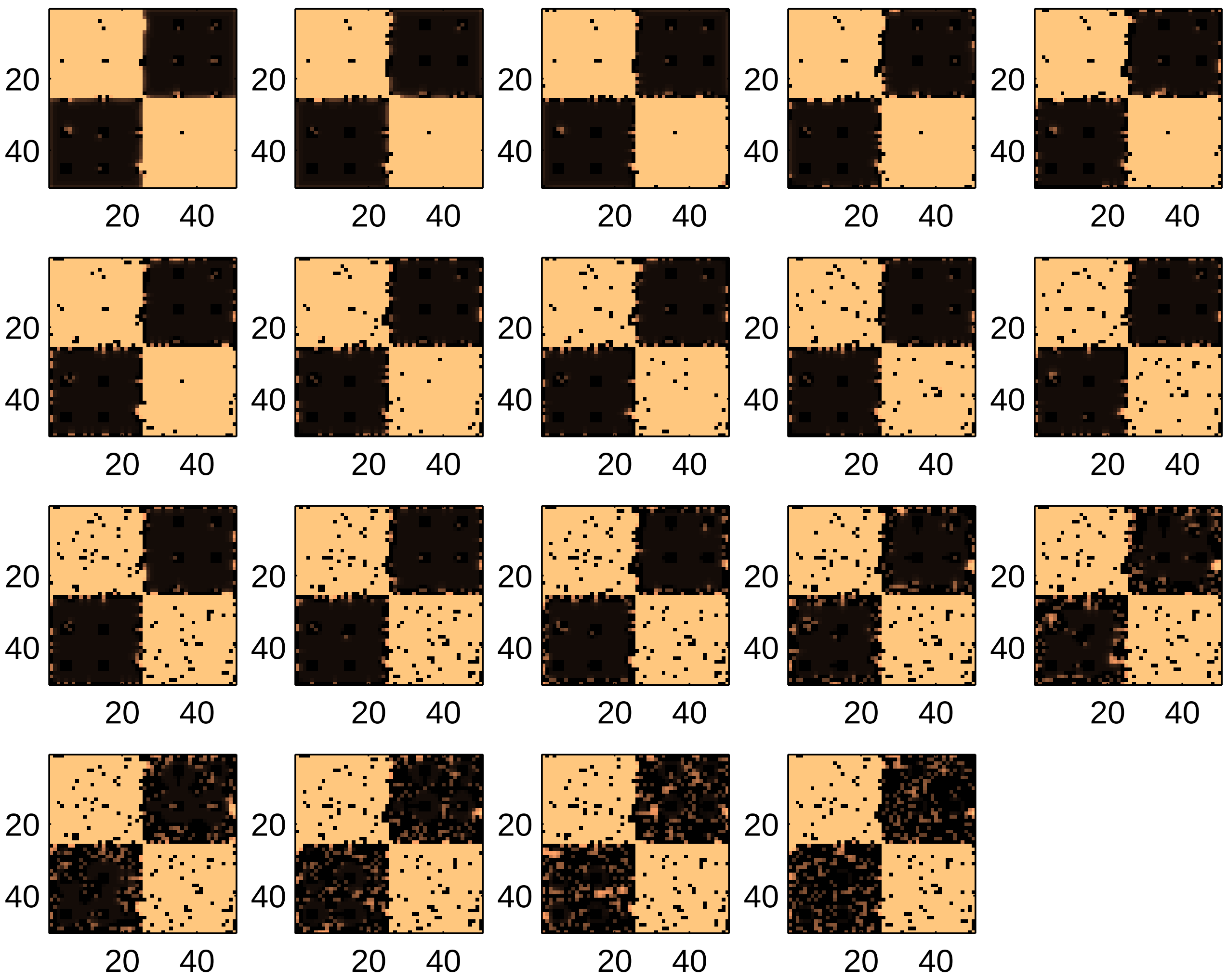
# Where do we sample?

Static sampling: A and O



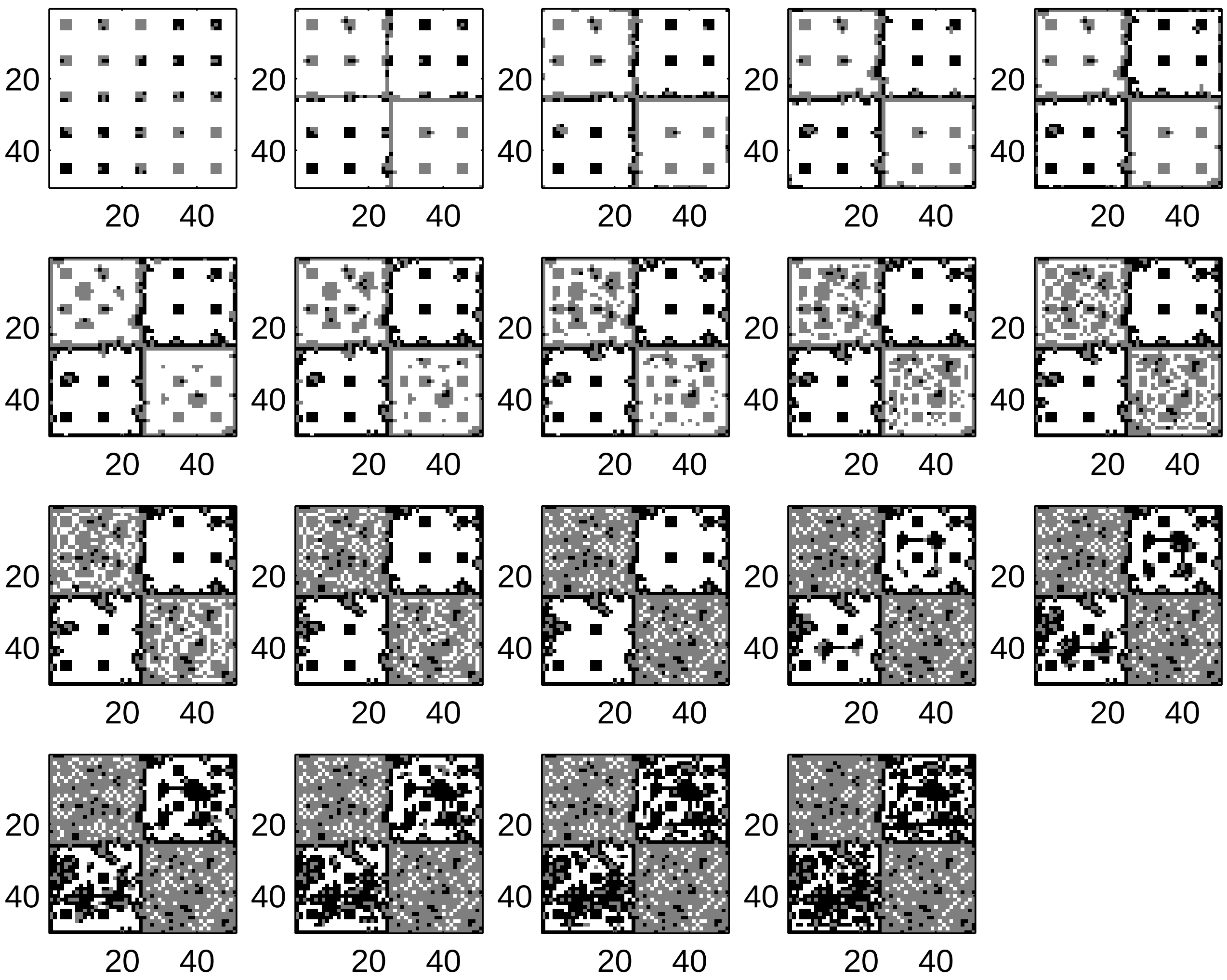
# Where do we sample?

Static sampling:marginals



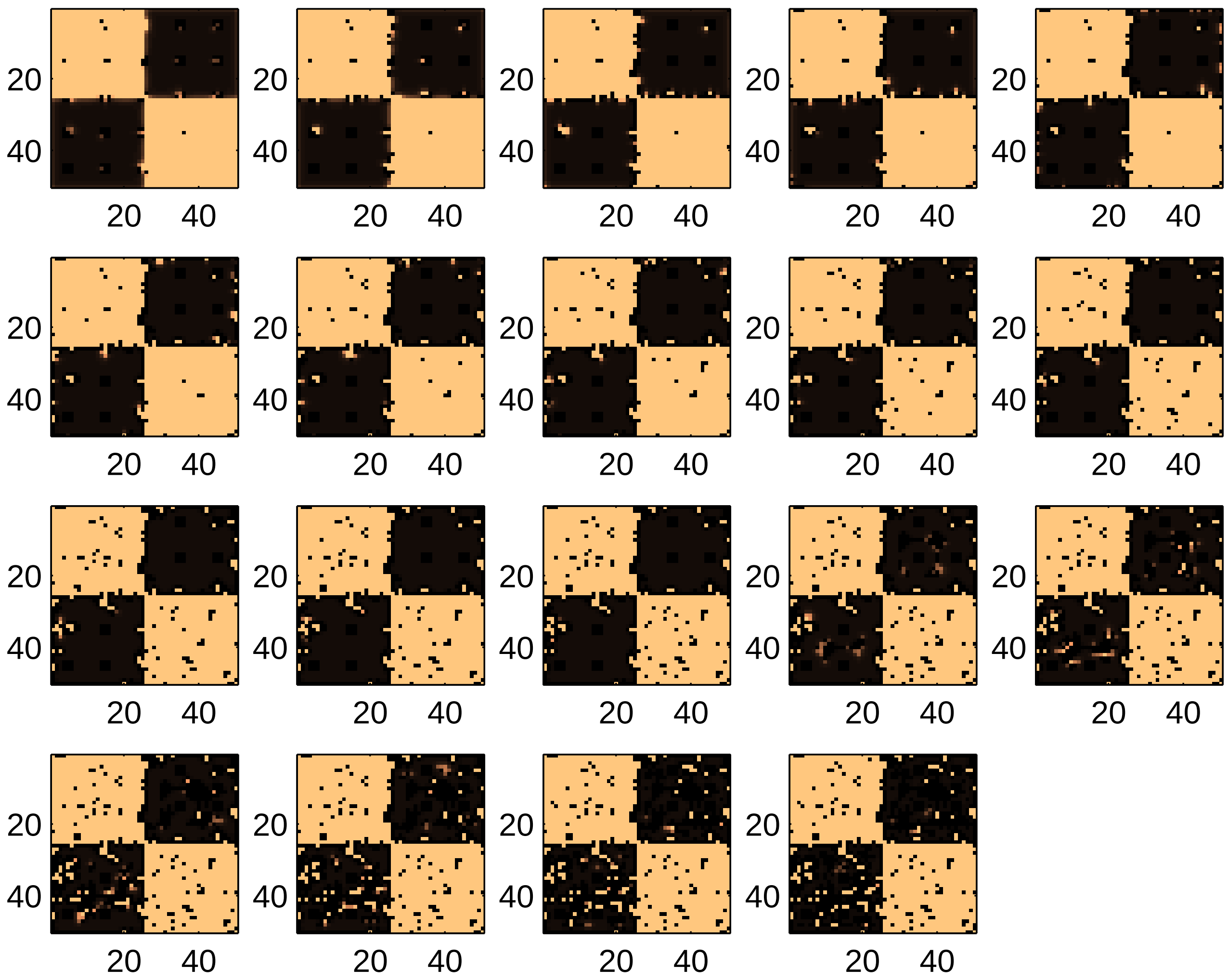
# Where do we sample?

Adaptive sampling: A and O (cumul)



# Where do we sample?

Adaptive sampling: marginals



# Where do we sample?

- No sampling in large empty areas
- Sampling preferably near detected occupied sites within low density areas
- If sampling ressources increase
  - SHS complete exploration until the whole area is covered
  - AHA can visit several times a site before extending exploration to another area