A NEW APPROACH TO THE INVENTORY PROBLEM IN LIFE CYCLE ASSESSMENT

A. MARVUGLIA, G. RIOS, R. WALLACE

Extended Abstract

Environmental life cycle inventory and assessment is a method which is used to quantify the environmental load and effects associated with a product, process, or service. The scientific literature offers a significant number of case studies in which the Life Cycle Assessment (LCA) methodology is applied. However, the computational structure of LCA, despite being of paramount importance for a deep comprehension of the validity and reliability of the results obtained in the inventory analysis phase, is rarely taken into account. LCA practitioners are generally interested in guidelines on which data to collect, which choices to make, and how to report assumptions and results and thus scientific publications generally try to answer to those needs. Practitioners generally limit themselves to commercial LCA software used to accomplish the study and rarely delve into the mathematical details.

The most widely used approach for the solution of the inventory problem is the matrix method. It determines the *inventory vector* related to a specific single or multifunctional unit process by solving a system of linear equations through the simple inversion of the so-called *technology matrix* $\mathbf{A}$ [1]. The first step for the calculation of the inventory vector of the studied process is represented by the determination of a *scaling vector* $\mathbf{s}$, through the resolution of the system of equations $\mathbf{As} = \mathbf{f}$. The matrix $\mathbf{A}$ represents the flows within the economic system, and the vector $\mathbf{f}$, called *final demand vector* or *external demand vector*, is an exogenously defined set of economic flows the operator requires that the system produce. After computing the scaling vector, it is possible to determine the inventory vector $\mathbf{g}$ (i.e. the vector containing all environmental flows associated with the reference process) by simply applying the equation $\mathbf{g} = \mathbf{Bs}$, where $\mathbf{B}$ (which is called *intervention matrix*) represents the environmental interventions of the system of unit processes. The computation of the inventory vector related to the studied process is the goal of the Life Cycle Inventory (LCI) phase.

When the technology matrix is square and invertible the solution of the problem is straightforward; however, when it is rectangular (with more rows than columns) the system is over-determined and a direct solution cannot be found. A rectangular data matrix is very common in LCI problems in different cases, such as:

- Cut-off of economic flows
- Multifunctional unit processes (processes delivering more than one valuable output)
- Closed-loop recycling

Some computational tricks are generally used to transform the technology matrix into a square and invertible one, but all of them introduce more uncertainty into the problem and lead to biased solutions [1]. Moreover, the technology matrix is the result of an estimation process or a measurement campaign and, as a consequence, it is always affected by several errors. In a system of equations, the presence of measurement errors in the vector of the constants terms $\mathbf{f}$ means that the system $\mathbf{As} = \mathbf{f}$ cannot be solved exactly, but instead one

solves the system of equations $\mathbf{As} \approx \mathbf{f}$. A common way to solve this problem is Ordinary Least Squares (OLS), in which one solves the system $\mathbf{As}_{OLS} = (\mathbf{f} + \Delta\mathbf{f})$, where $\Delta\mathbf{f}$ is the residual error vector corresponding to a perturbation in $\mathbf{f}$. The OLS solution vector $\mathbf{s}_{OLS}$ is chosen to minimize the squared Euclidean norm of the *discrepancy vector* $||\mathbf{As}_{OLS} - \mathbf{f}||_2^2$. However, it is well known that OLS leads to a biased estimate of the solution vector [2].

Another approach that yields a consistent estimator is Total Least Squares (TLS), which is a linear parameter estimation technique that has been devised to compensate for data errors [3, 4]. It is a natural generalization of OLS and it is used when data in both $\mathbf{A}$ and $\mathbf{f}$ are allowed to be perturbed.

A particular case of TLS is the so-called Data Least Squares (DLS) problem in which the error is assumed to lie only in the data matrix leading to the system $(\mathbf{A} + \Delta\mathbf{A})\mathbf{s}_{DLS} = \mathbf{f}$, where $\Delta\mathbf{A}$ is the noise portion of the matrix $\mathbf{A}$. The DLS solution vector is chosen so that the norm of $\Delta\mathbf{A}$ is minimized [5].

The authors present the three aforementioned least squares techniques in a case study and highlight the differences in the results. Further, we propose an extension of the methodology by introducing discrete constraints on the technology matrix or possibly on the discrepancy vector. Introduction of constraint programming (CP) into the analysis allows us to investigate possible variations in energy and material flows supporting choice between different scenarios involving mutually exclusive alternatives. Although never applied in direct connection with LCA, CP methodology has been applied to assess the compliance of a product with an *eco-label* [7]. A related approach using linear programming has been applied in LCA [6], but this technique is not suitable for handling discrete constraints such as scenarios with mutually exclusive choices. The CP approach can easily overcome this drawback and play a significant role in identification of opportunities for environmental improvements in a product system. This enables the expert to test a set of alternatives for system improvements rather than a single optimum, making it possible to find the Best Practicable Environmental Options (BPEO) *not entailing excessive costs.*

References

[1] Heijungs R, Suh S. The computational structure of life cycle assessment. Kluwer Academic Publihers, Dordrecht, 2002, Series: Eco-Efficiency in Industry and Science, Vol. 11.

[2] Levin MJ. Estimation of a system pulse transfer function in the presence of noise. IEEE Transactions on Automatic Control 1964; 9:229-235.

[3] Golub GH, Van Loan CF. An analysis of the total least squares problem. SIAM Journal on Numerical Analysis 1980; 17:883-893.

[4] Van Huffel S, Vandewalle J. The Total Least Squares Problem: Computational Aspects and Analysis. SIAM: Philadelphia, 1991.

[5] DeGroat RD, Dowling EM. The Data Least Squares Problem and Channel Equalization. IEEE Transactions on Signal Processing 1993; 41(1):407-411.

[6] Azapagic A, Clift R. Linear Programming as a Tool in Life Cycle Assessment. International Journal of Life Cycle Assessment 1998; 3(6): 305-316.

[7] Houe R, Grabot, G. Assessing the compliance of a product with an eco-label: From standards to constraints. International Journal of Production Economics 2009; 121: 2138.

E-mail address: `{a,g,r}.{marvuglia,rios,wallace}@4c.ucc.ie`

Cork Constraint Computation Centre, University College Cork, Ireland